SOUTH CAROLINA
STATE DEPARTMENT
OF EDUCATION

Visual and Concrete Models for Division and Multiplication of Decimals as Set Forth in the Common Core State Standards for Mathematics

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Today's Learning Progression:

- ✓ **Introduce base ten blocks as decimals**
- ✓ **Represent decimals with base ten blocks**
- ✓ **Use base ten blocks to represent decimal lengths**
- ✓ **Form arrays with decimal side lengths using base ten blocks (decimal multiplication)**
- ✓ **Relate decimal multiplication to decimal division**
- ✓ **Create concrete and pictorial models for division of decimals**

Use the CCSSM portion of the Standards Overview and the 5th grade CCSSM page at a glance handout to discuss with your neighbor responses to the following:

- **What are the CCSSM expectations with regard to multiplication and division of decimals?**

- **With regard to decimals, what else does CCSSM expect at your grade?**

Use the 2007 portion of the Standards Overview handout to reflect on the following based on your transition status:

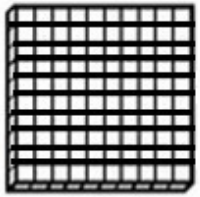
- **What student gaps in decimal-related learning may need to be addressed as a result of when/how your school transitioned to CCSSM?**

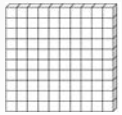
Students need to be <u>introduced</u> to the base ten blocks from a decimal perspective

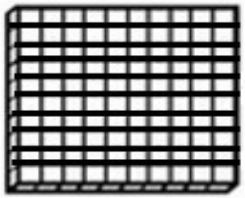
Take out your base ten blocks

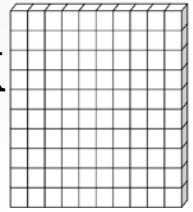
How many pennies in a dollar?

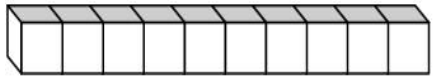
- If you could use only one block to represent a dollar, which one would you use and why?

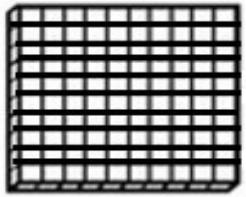
Whole 	Rod	Unit
1 whole $1.00		

So, if this block  represents 1 whole or $1.00 which block would you use to represent 1 penny and why?

Whole 	Rods 10-block	Hundredths
1 whole $1.00		1 penny $.01 1/100 of a dollar .01

So if this block  represents 1 whole or $1.00

and this block represents 1/100 or .01 of that whole, what does this block  represent and how do you know?

Whole 	Tenths	Hundredths
1 whole $1.00	1 dime $.10 1/10 of a dollar .1 or .10	1 penny $.01 1/100 of a dollar .01

Students need <u>practice</u>

using base ten blocks to

represent decimals.

Let’s use our blocks to represent amounts

- ***Take out your decimal place value mat***

- **Use the fewest number of blocks to represent $1.48 on the mat.**
- **Rather than drawing the entire block, how might we represent this in picture form?**
- **Write the amount three ways and draw a pictorial model on your paper.**

Let’s use our blocks to represent amounts

- **Use the fewest number of blocks to represent the price on your mat.**
- **Write the amount three ways and draw a pictorial model/representation on your paper.**

Let’s use our blocks to represent amounts

- **Use the fewest number of blocks to represent the price on your mat.**
- **Write the amount three ways and draw a pictorial model/representation on your paper.**

Let’s use our blocks to represent amounts

- **Use the fewest number of blocks to represent 2.51 on your mat.**
- **Write the amount three ways and draw a pictorial model/representation on your paper.**

- **Use the fewest number of blocks to represent 1.04 on your mat.**
- **Write the amount three ways and draw a pictorial model/representation on your paper.**

Students need practice using base ten blocks to represent <u>decimal lengths</u>

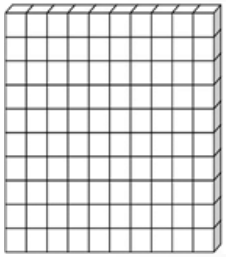
Students may experience difficulty transitioning from thinking about the blocks in terms of money or "plain" decimal <u>quantities</u> to thinking about the blocks in terms of decimal <u>square units or lengths</u>.

Prior to inducing decimal lengths using base ten blocks, keep in mind. . . .

CCSSM requires that 2nd grade students estimate and measure in centimeter increments.

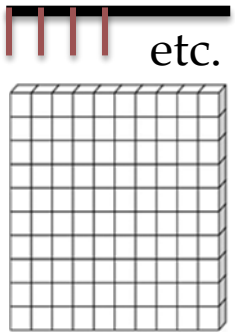
However, review/introduction of using measuring devices with centimeter increments may be necessary – depending on where you are with transition to CCSSM and the mathematical strength of your students.

Using base ten blocks to represent decimal lengths. . .

When we were representing money and "plain" decimals with base ten blocks, what did we say is the value of this block  **?**

So, as we begin to think about drawing decimal lengths using base ten blocks, this will be one square unit. The length and the width of this block are each one whole unit long.

On a piece of paper use the whole square unit block to draw a horizontal line that is 1 whole unit long.



Use the "grid" on the whole unit block to subdivide that line into 10 equal parts.

Talk with your neighbor – what is the length of the space between each of the 10 parts and how do you know?

Which block represents .1 square unit and why?

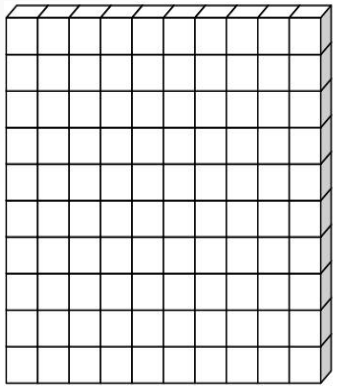
Students may experience difficulty transitioning from thinking about the blocks in terms of money or "plain" decimal <u>quantities</u> to thinking about the blocks in terms of decimal <u>square units or lengths</u>.

On a piece of paper use your whole unit (unit square) base ten block to draw a length of .3 unit and one of .7 unit. Use a number of the .1 square unit pieces to check the lengths.

Talk with your neighbor – what could you do if you wanted to draw a line longer than 1 unit – say 1.4 units for example?

Use your whole square block to draw lines of the following lengths, then use the .1 square blocks and then a centimeter ruler to verify the lengths:
1.6 units long
1.3 units long

If I asked you to measure the width of this block

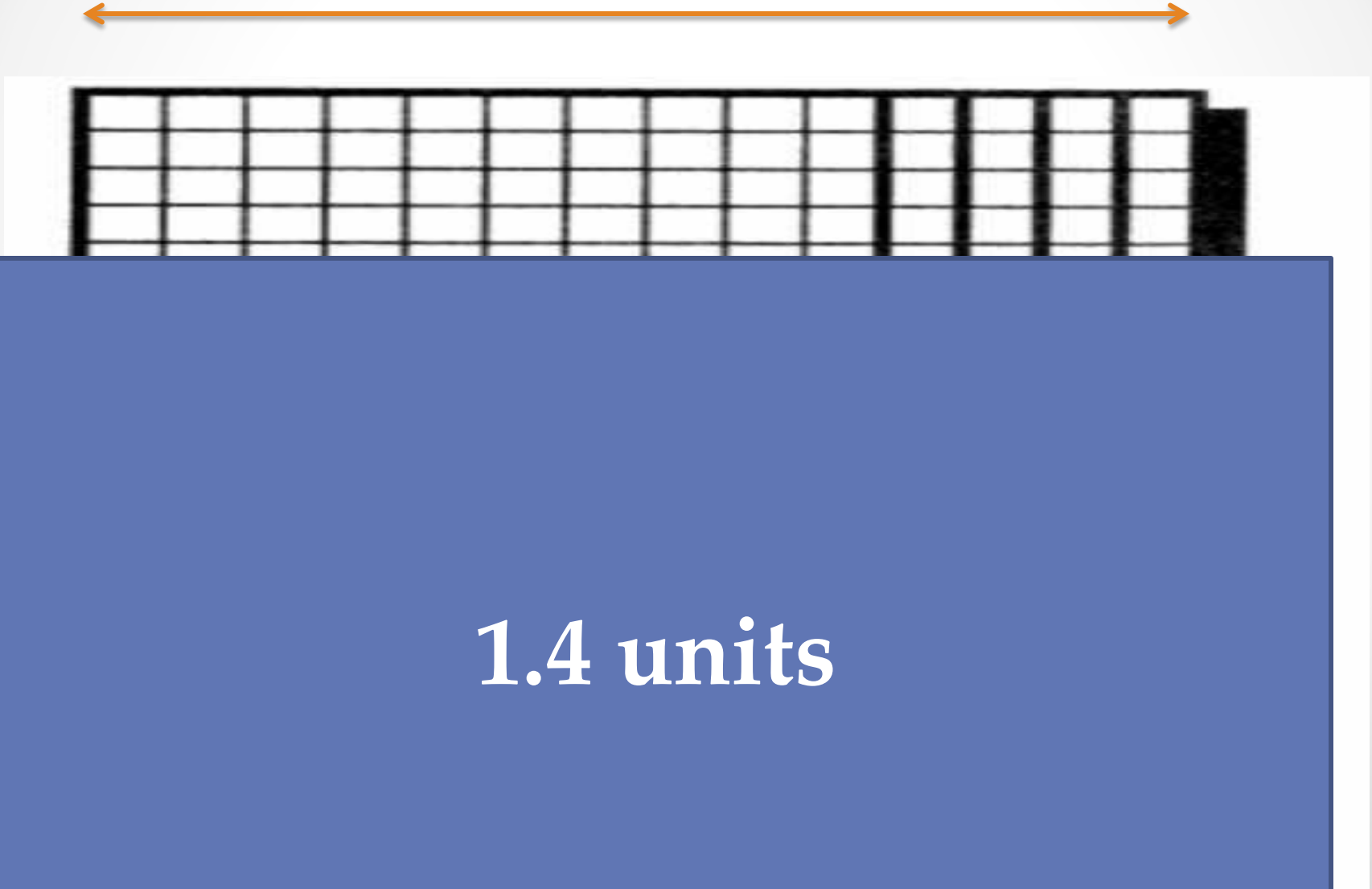


show me what you'd measure.

If I asked you to measure the length. . .

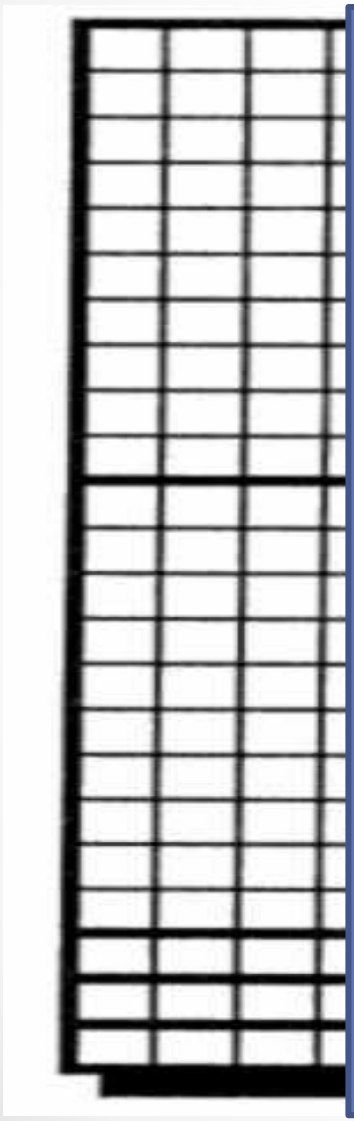
In this case, does it matter what we call length and what we call width?

What is the width of this partial picture? Explain your answer to your neighbor.



1.4 units

What is the length of this partial picture? Explain your answer to your neighbor.

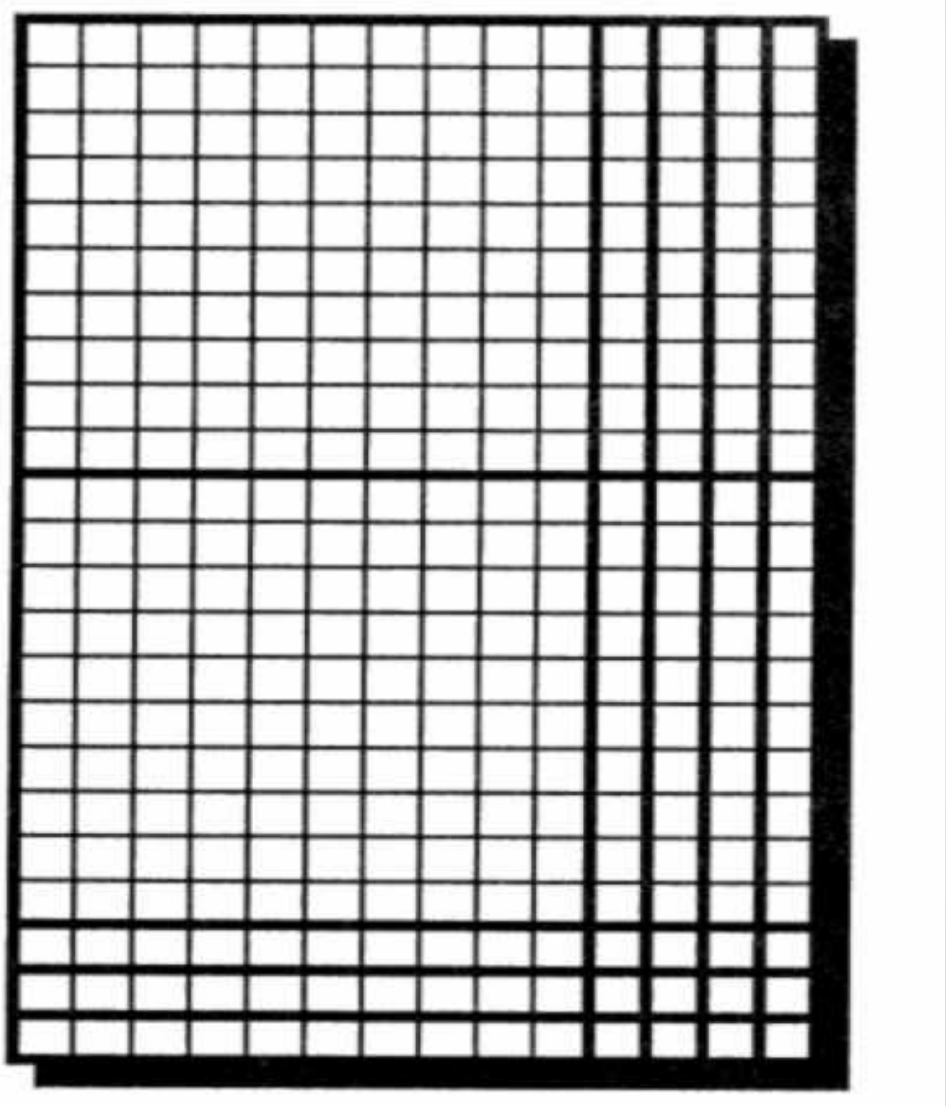


2.3 units

Now, here's the whole picture. Based on what you just determined, what are the dimensions of this array?

1.4 units x 2.3 units

Use base ten blocks to form this array.



Take out the sheet with the two large blank rectangles.

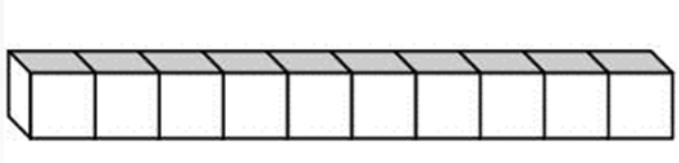
Use the fewest number (minimal collection) of base ten blocks to fill in the large rectangle. What are the dimensions of the rectangle/array?

1.6 units x 1.2 units

Use the fewest number (minimal collection) of base ten blocks to fill in the small rectangle. What are the dimensions of the rectangle/array?

1.3 units x .8 unit

What would the dimensions be if we used only this block

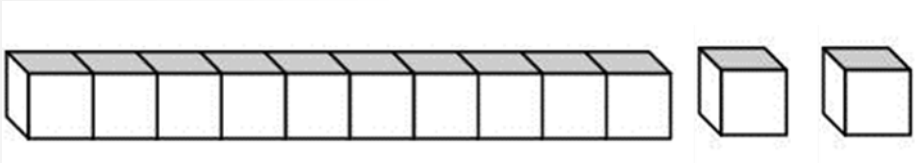


and why?

Talk with your neighbor.

1 x .1

What would the dimensions be if these blocks were all touching and why?

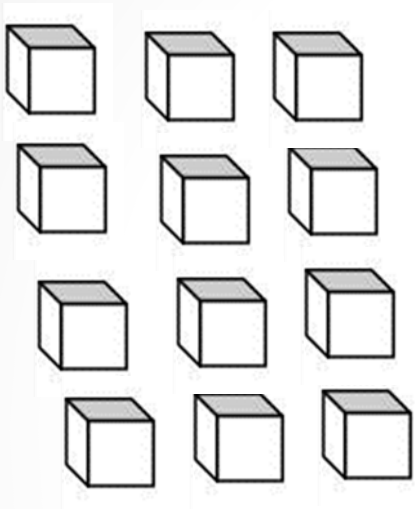


Talk with your neighbor. **1.2 x .1**

Write 1.2 x .1 = .12 on paper and draw the model.

What do you notice about the product and the area of the model?

What would the dimensions be if these blocks were all touching and why? – Build it the way it looks but with blocks touching.

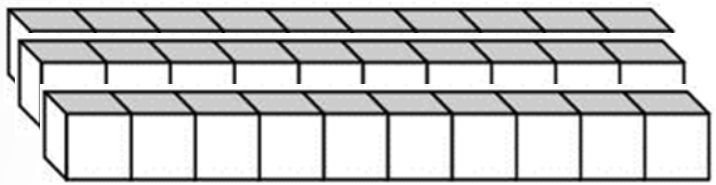


Talk with your neighbor.

Write .3 x .4 = .12 on paper and draw the model.

What do you notice about the product and the area of the model?

Use a minimal number of blocks to form an <u>array</u> that measures 1unit x .3 unit.



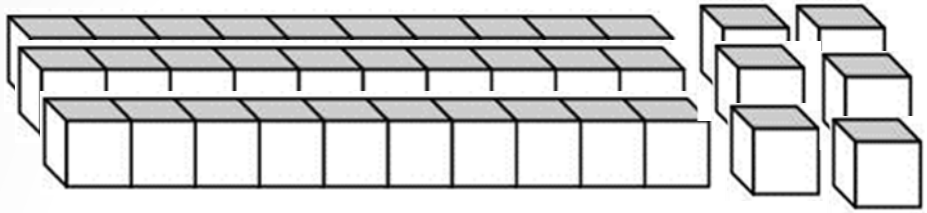
Imagine blocks are touching.

Write 1 x .3 = .3 on paper and draw the model.

What do you notice about the product and the number of tenth strips in the array?

What is the relationship between the number of "small" cubes in the array (product/area) and the number of "small" cubes in the whole unit?

Use your blocks to form an array that measures 1.2 units x .3 unit.



Imagine blocks are touching.

Write 1.2 x .3 = .36 on paper and draw the model.

What do you notice about the place value of each digit in the product and the number and type of each block in the array?

What is the relationship between the number of "small" cubes in the array (product) and the number of "small" cubes in the whole unit?

Use your blocks to form a model that measures 1.2 units x 1.3 unit.

What is the equation represented by that model – <u>Determine the product from the model – not multiplication</u>.

1.2 x 1.3 = 1.56 - Draw the model.

What do you notice about the place value of the digits in the product and the number of each type block in the model?

Use your blocks to form a model that measures .2 x 3.7

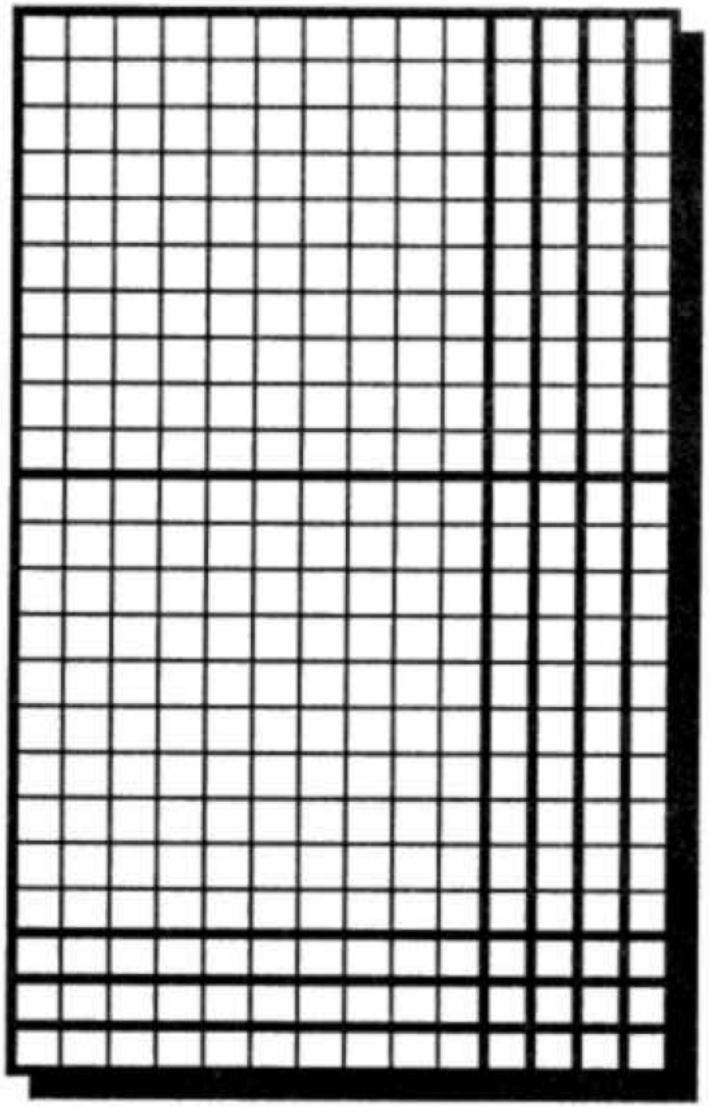
What is the equation represented by that model - <u>Determine the product from the model – not multiplication.</u>

.2 x 3.7 = .74 - Draw the model

What do you notice about the place value of the digits in the product and the number of each type block in the model?

How does this differ from the other models we formed? The 14 small cubes can be traded to get another ten with 4 hundredths left over. Thus the minimal collection of 7 tenths and 4 hundredths or .74.

This is the model we used when we first starting talking about decimal lengths. The dimensions are 1.4 x 2.3.



Find the area without multiplying and explain your reasoning.

Write an equation to represent the model.

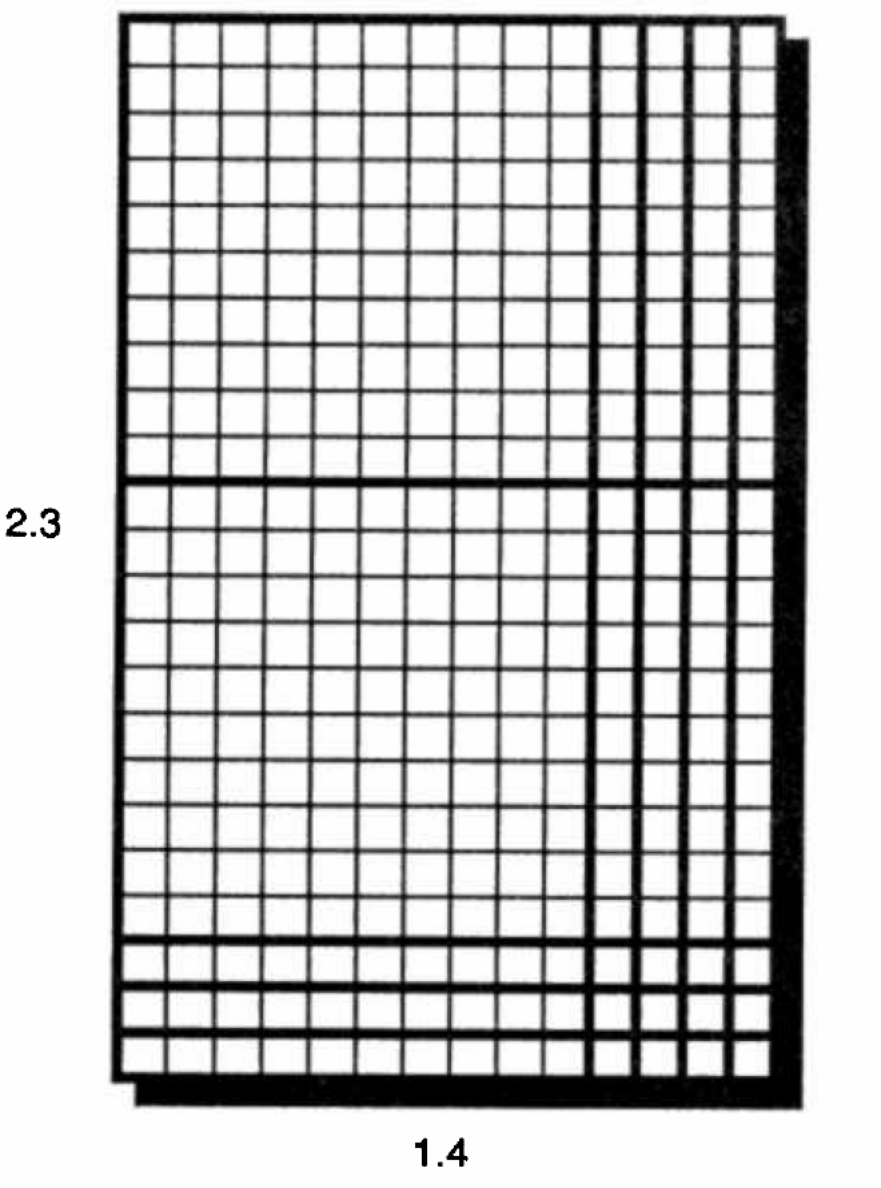
1.4 x 2.3 = 3.22 Draw the model

Look at all the products you have written so far. What is the relationship between the location of the decimal in the product and where it is in each of the factors? Explain.

What questions or comments do you have before we transition to dividing?

What is the relationship between multiplication and division?

5 x 8 = 40 OR
1.4 x 2.3 = 3.22



2.99 ÷ 2.3 =

What is the dividend and what is the divisor?

If you were to form a model to solve this, which number would be the area of the model? Why?

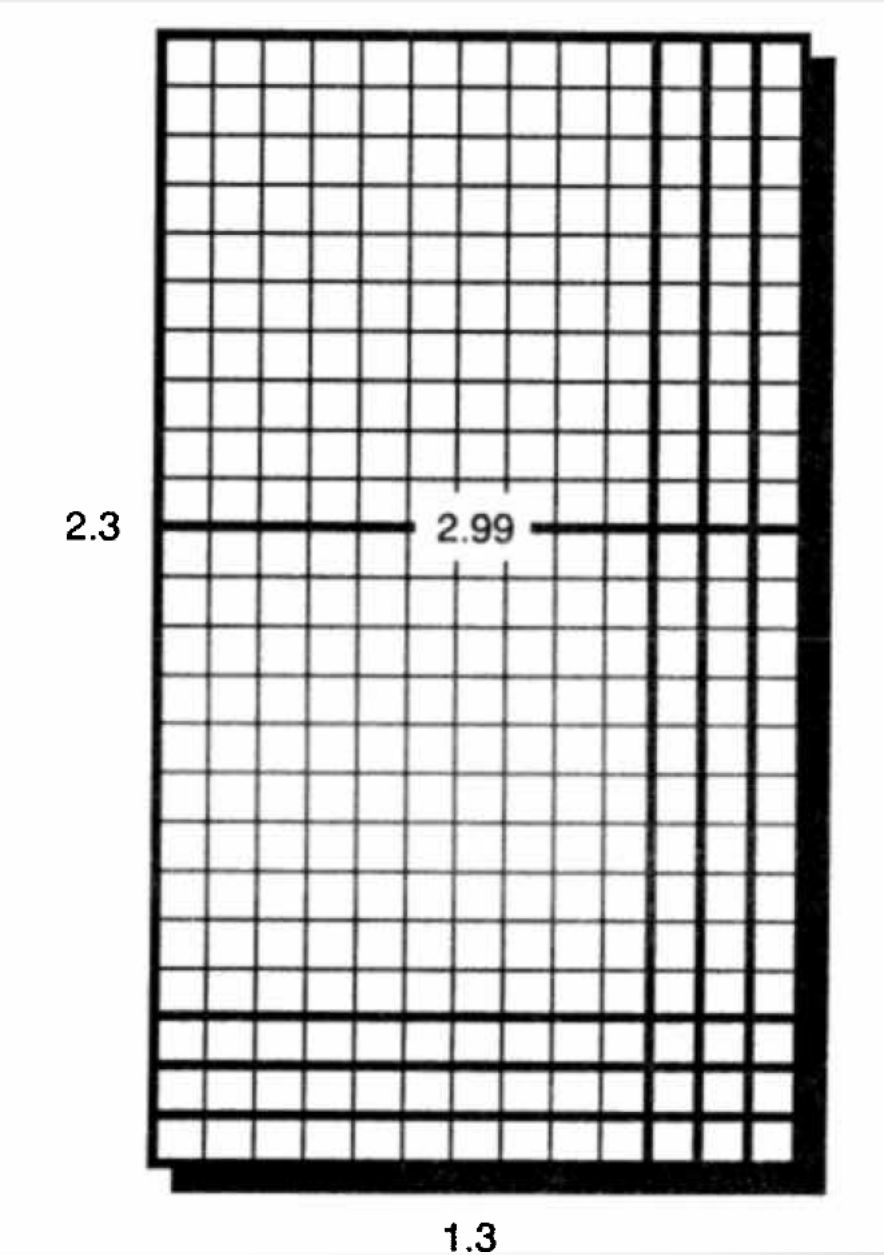
What will be one of the dimensions (side lengths) and how do you know?

2.99 ÷ 2.3 =

Based on the area (dividend) and without thinking about trading, what blocks will you need to form a model with that area?

2 – ones; 9 tenths; and 9 hundredths

Form the model and write a division equation.



Draw the model. Write a multiplication equation to represent the model.

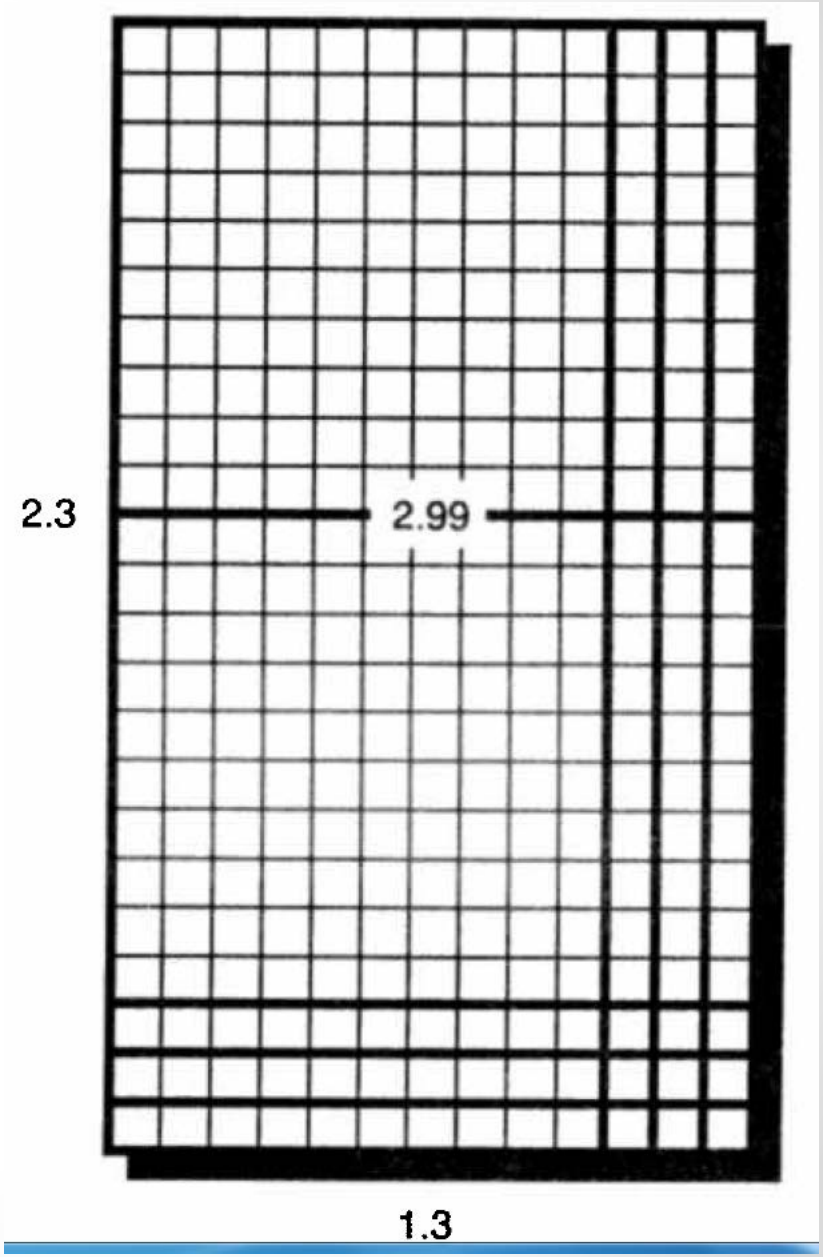
Now write them again using the and the "X" symbols/forms.

Explain to your neighbor the relationship between the two equations.

Have your neighbor explain to you how the two equations relate to the model.

Prepare to share.

(Subtraction)



3.08 ÷ 1.4 =

What is the dividend and what is the divisor?

If you were to form a model to solve this, which number would be the area of the model? Why?

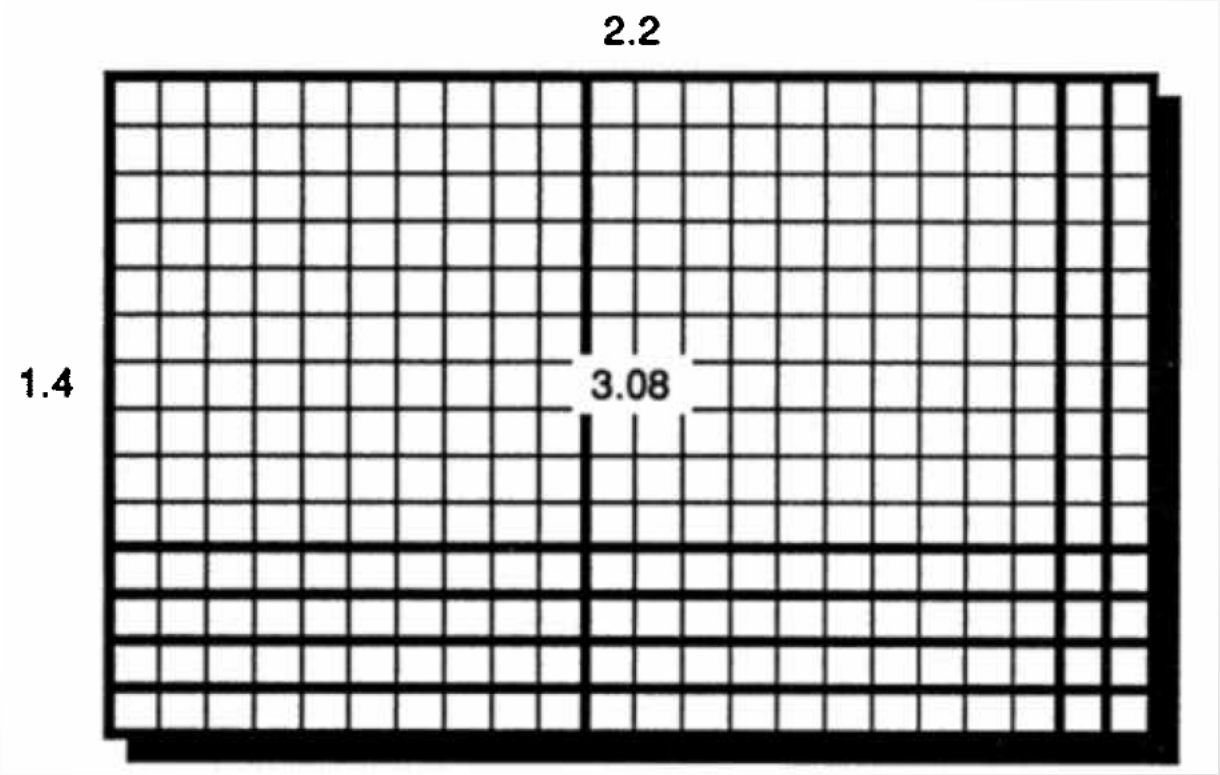
What will be one of the side lengths (dimensions) and how do you know?

3.08 ÷ 1.4 =

Based on the area(dividend) and without thinking about trading, what blocks will you need to form a model with that area?

3 – ones and
8 hundredths

Form the model. Write a division equation to represent the model.



Draw the model. Write a multiplication equation to represent the model.

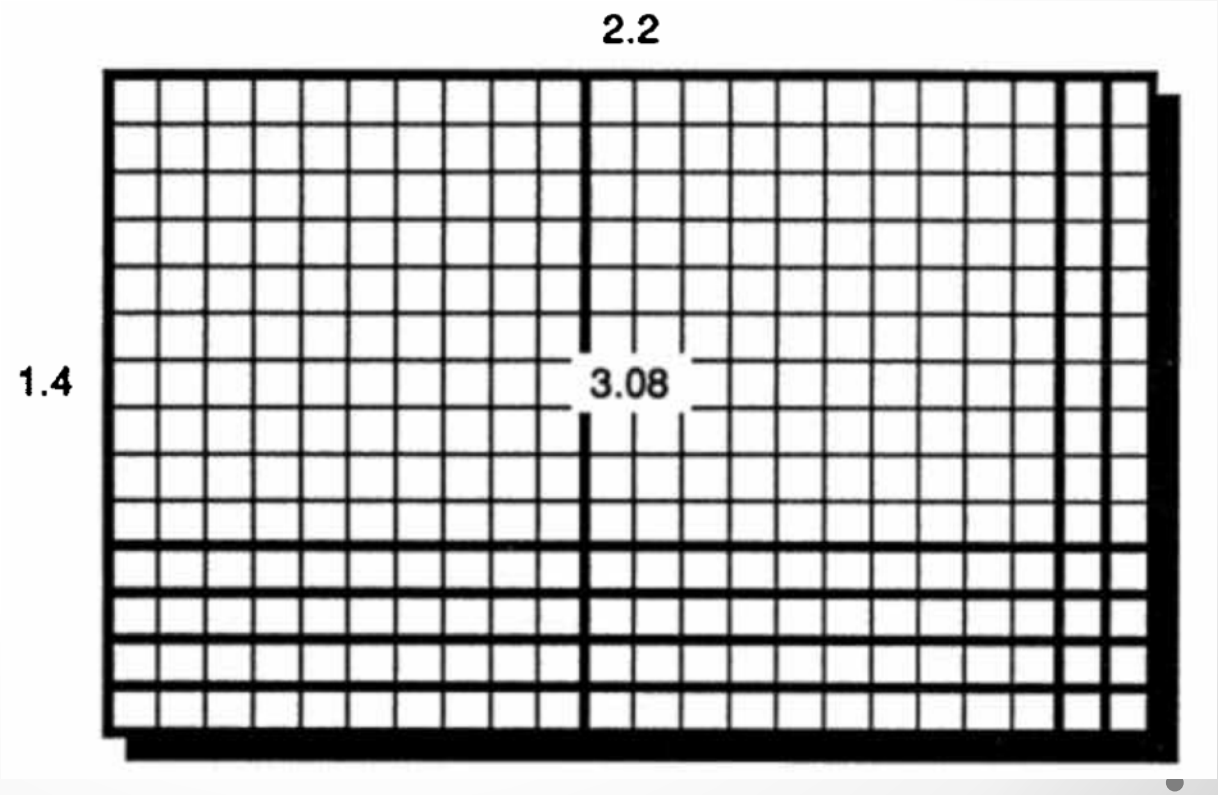
Now write them again using the and the "X" symbols/forms.

Explain to your neighbor the relationship between the two equations.

Have your neighbor explain to you how the two equations relate to the model.

Prepare to share.

Subtract?



How do the cognitive requirements for solving the first division problem differ from the second division problem?

Talk with your neighbor.

Be prepared to share.

2.04 ÷ 1.2 =

What is the dividend and what is the divisor?

If you were to form a model to solve this, which number would be the area? Why?

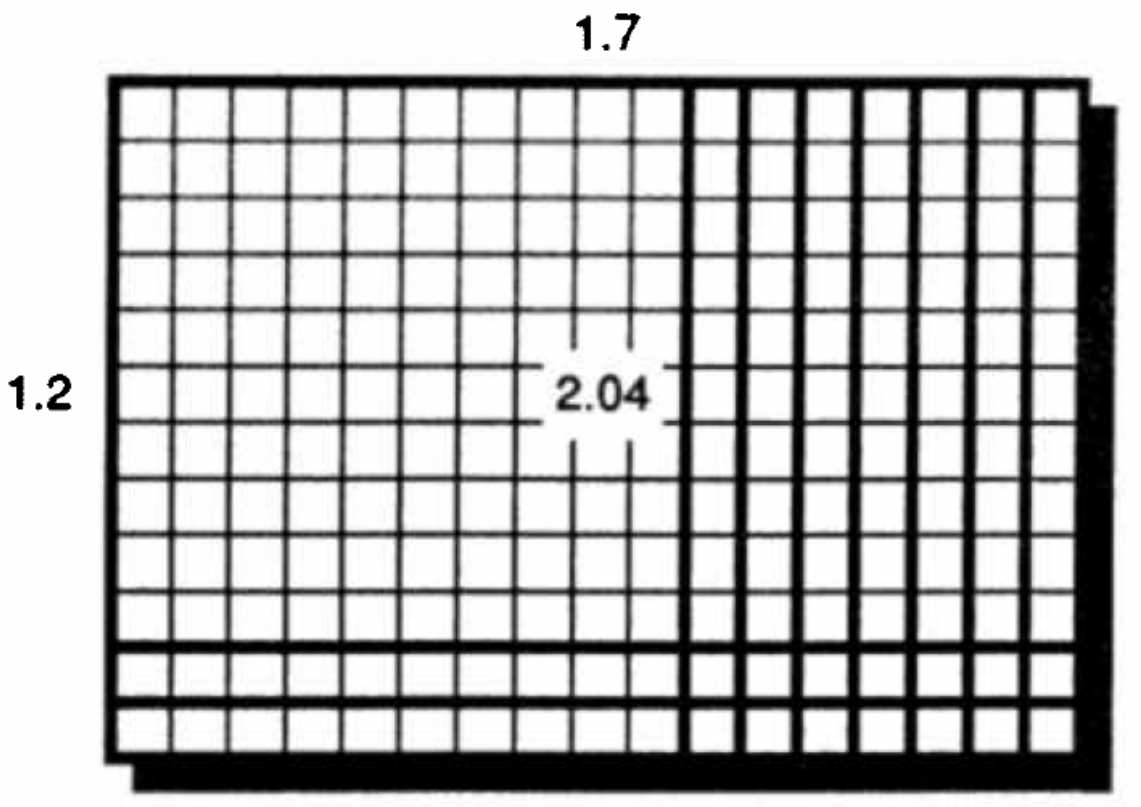
What will be one of the side lengths (dimensions)? How do you know?

2.04 ÷ 1.2 =

Based on the area(dividend) and without thinking about trading, what blocks will you need to form a model with that area?

2 – ones; and
4 hundredths

Form the model.
Write a division equation to represent the model.



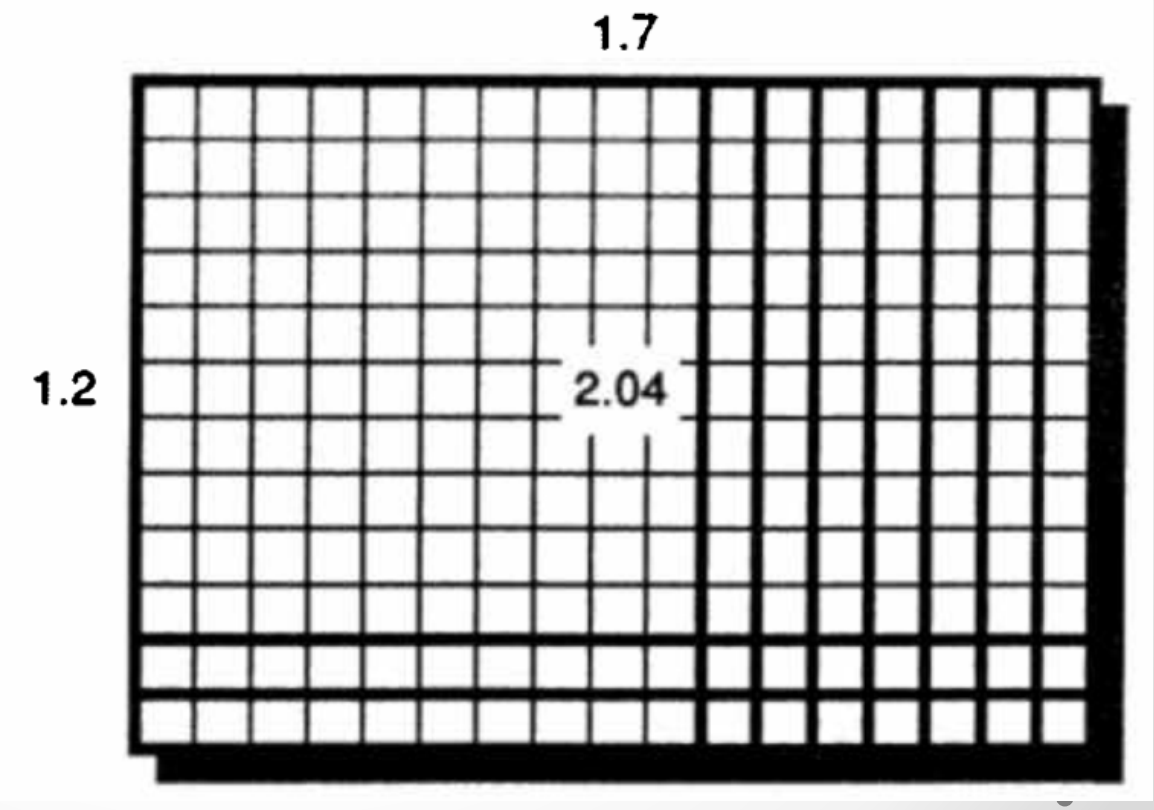
Draw the model. Write a multiplication equation to represent the model.

Now write them again using the and the "X" symbols/forms.

Explain to your neighbor the relationship between the two equations.

Have your neighbor explain to you how the two equations relate to the model.

Prepare to share.



.77 ÷ 1.1 =

What is the dividend and what is the divisor?

If you were to form a model to solve this, which number would be the area of the model? Why?

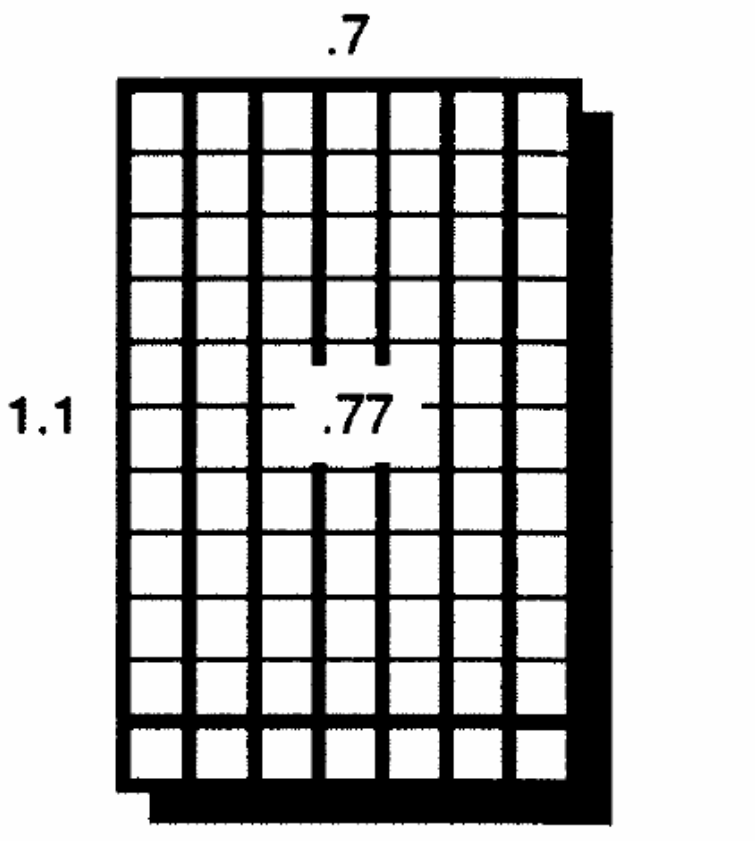
What would be one of the side lengths (dimensions) and how do you know?

.77 ÷ 1.1 =

Based on the area(dividend) and without thinking about trading, what blocks will you need to form a model with that area?

7 – tenths; and
7 hundredths

Form the model. Write a division equation to represent the model.

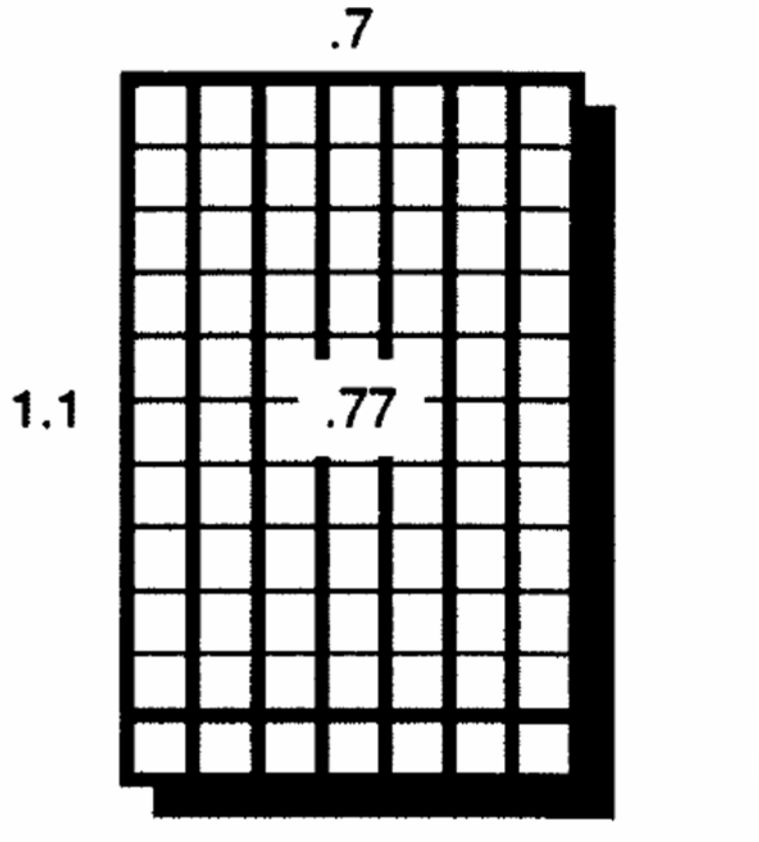


Write a multiplication equation to represent the model.
Now write them again using the and the "X" symbols/forms.

Explain to your neighbor the relationship between the two equations.

Have your neighbor explain to you how the two equations relate to the model.

Prepare to share.



How do the cognitive requirements for solving the last two division problems differ?

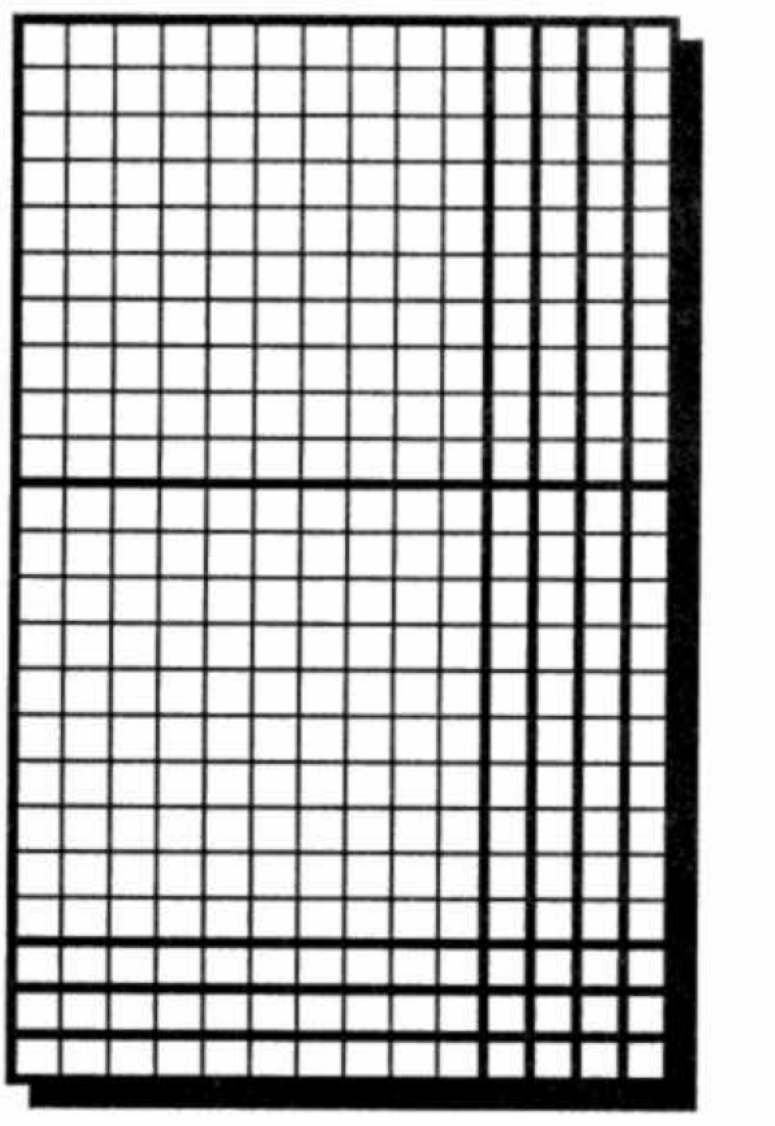
Talk with your neighbor.

Be prepared to share.

REPEAT---

Do you see a relationship between where the decimal is placed in the product and where it is in each of the factors? Explain.

How does where you put the decimal in the product relate to the place value of the two factors?



Tenths x tenths = hundredths, etc.

Look at the division equations you wrote.

The question is where do we put the decimal in the quotient if we don't have a model?

One important thing to remember when dividing – whole numbers or decimals – neatness counts – it is important to line up digits in the quotient with the last digit of the number into which you are dividing.

When dividing decimals, it is easier to divide by whole numbers; estimate; etc.

Problems in context!!!!

My Contact

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Feedback and Certificate of Participation

https://ed.sc.gov/Survey/index.php?sid=55561&lang=en